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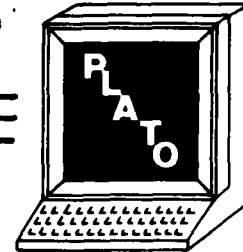81—1·
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PSYCHOMETRIC APPROACH TO ERROR ANALYSIS ON RESPONSE PATTERNS OF ACHIEVEMENT TESTS

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COMPUTERIZED ADAPTIVE TESTING AND MEASUREMENT   **RESEARCH REPORT 80-3**

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

-This study is an attempt to improve the quality of computerized adaptive testing as an integral part of instruction. An adaptive achievement test for teaching signed-numbers operations was implemented along with a computerized routing system on the PLATO system in 1977. A close investigation of the student's performance-scores on the posttest of this program led us to believe that a deeper level of considerations, not just the data

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scored right or wrong, in measuring students performance would be needed
in future computerized tests. Diagnosing the misconceptions possessed by
students is important not only for increasing efficiency of learning
activities but also to score test-items properly. Some problems
could be ~~got right~~ by a wrong rule of operation. These may be called
"false corrects." It is shown in this work that in some cases as
many as 27 of the 64 test items need to be adjusted from right to
wrong if we are to discredit "false corrects". Finding all types of
wrong rules of operation associated with a particular teaching method
for integer operations was tried by performing error analysis on
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Our approach to performing error analysis on the conventional tests is
to generate "error vectors" from item-responses, instead of generating
responses from a "process network" as some researchers have done. By
checking error vectors generated from a particular set of items, a
consistent error or wrong operation can be diagnosed. The system of
"error vectors", which is equivalent in power to the "process network",
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applied to adaptive achievement testing, the validity of scoring would
be greatly improved.

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ABSTRACT

This study is an attempt to improve the quality of computerized adaptive testing as an integral part of instruction. An adaptive achievement test for teaching signed-numbers operations was implemented along with a computerized routing system on the PLATO system in 1977. A close investigation of the students' performance-scores on the post-test of this program led us to believe that a deeper level of consider-ations, not just the data scored right or wrong, in measuring students performance would be needed in future computerized tests. Diagnosing the misconceptions possessed by students is important not only for increasing efficiency of learning activities but also to score test-items properly. Some problems could be got right by a wrong rule of operation. These may be called "false corrects." It is shown in this work that in some cases as many as 27 of the 64 test items need to be adjusted from right to wrong if we are to discredit "false corrects". Finding all types of wrong rules of operation associated with a particular teaching method for integer operations was tried by performing error analysis on some paper-and-pencil as well as on-line conventional tests.

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a conventional test, as well as to develop an adaptive test for diagnosing the misconceptions possesed by a student. If this procedure were applied to adaptive achievement testing, the validity of scoring would be greatly improved.

PSYCHOMETRIC APPROACH TO ERROR ANALYSIS ON

RESPONSE PATTERNS OF ACHIEVEMENT TESTS

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and

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INTRODUCTION

Birenbaum & Tatsuoka (1980) have identified various erroneous
rules of operations in signed-number arithmetic. Many researchers
have investigated errors observed in classroom teaching or performance
data from achievement tests administered in conjunction with instruction.
Error analysis had mainly been utilized in teaching and instructional
areas until Brown & Burton (1978) introduced "BUGGY", an adaptive
testing system by which students' misconceptions can be diagnosed. The
computer algorithm of BUGGY is based on a (universal) cognitive process
network of addition and subtraction problems of whole numbers. In
order to construct such a network, error analysis of the subject area
must be thoroughly conducted first so that the mechanics of the mental
processing in achieving a given task in the area becomes clear. Theories
of cognitive processes have an important role in determining and
constructing a process network of problem solving tasks.

Our approach described in this study is somewhat different from the BUGGY approach. Constructing a universal process network of any given task, in our case the computational skills of signed-number operations, is very difficult. Moreover, our experimental study investigating the effect of different instructional backgrounds on error types indicates that there are significant differences in mental processing to arrive at the answers among students who studied different instructions. Thus, the outcome of error analysis resulted in discovering quite a difference in the types of errors committed by each group. This preliminary result of the experiment suggests that there might not be a unique universal process network which is applicable to all error types and attributable as the source of each error. Errors depend on different instructional methods in which the conceptual framewrok of the problem is treated by different approaches. Considering the above-mentioned difficulties, a method of diagnosing misconceptions which is more adjustable to a change of instructional method is introduced.

Our approach of using "error vectors" generates a binary vector of erroneous and correct actions taken by the student. Elements of an error vector comprise all possible operations in doing a problem. Thus, for an addition problem involving signed numbers (positive and negative. integers), a three-dimensional binary vector represents the actions taken (=1) or not taken (=0) in getting the absolute value of the answer. The successive elements represent the following actions: (1) Doing the correct operation; (2) adding the absolute values of two numbers; (3) subtracting them. Separately, a six-dimensional binary vector

does the same for the action taken or not taken in getting the sign of the answer. The matrix product of these two vectors yields a set of 18 "events", each being a combination of two possible errors associated with the operations of taking absolute values and signs in the answer.

When the two vectors for each of several problems have been generated from each student's responses, the elementwise multiplication of all the vectors of each type, the "absolute value" vector and the "sign" vector, respectively, produces a pair of error vectors for the entire set of problems. If the student behaved consistently, applying his/her rule throughout the problems, then the final error vectors will contain just one 1 and all other elements will become zeros. Therefore the matrix product of the two dimensions, representing the operations of absolute values and signs to the answer will uniquely determine the type of misconseption, if any.

The errors diagnosed by the vector approach were compared with the results obtained from two different approaches: One is examining a response pattern of the answers to the problems as a whole and deducing the student's rule of operations intuitively (Birenbaum & Tatsuoka, 1980; Neches, 1978); the other is interviewing a few students and asking them to tell us their rules of operation and reasoning. It was confirmed that the error vector system works equally well as the other approaches.

In this paper, the error vector system for addition problems, which is based on the instruction given by the teachers at Urbana Junior High School in their classes, will be introduced and then the

error vector system for subtraction problems will be introduced and discussed briefly.

An adaptive test capable of diagnosing the student's misconceptions in the addition problems was programmed on the PLATO system (a Computer-based Education System at the University of Illinois) and it was tried with 180 seventh graders in January, 1980. Evaluation of the results indicated an interesting change in the student's mental processing activity that has probably never been observed in the traditional scoring procedure of any tests. Moreover, it was observed that quite a number of students switched the rules of operations from one to another -- sometimes from the right rule to a wrong rule, sometimes the other way around. By observing these shifts among different error types, one can conclude that there are certain errors which can eventually be converted to the right rule of operation without having to repeat the same instruction after the test, but some errors will never behave like these innocuous errors. This study suggests that it is possible to develop a quantitative model of scoring so that each error type will be assigned a real number which is an indicator of the seriousness of the error. Some serious errors are due to musunderstanding of an important concept while others are based on a simple mistake which can be corrected very easily.

ERROR VECTORS FOR ADDITION PROBLEMS
OF SIGNED-NUMBER COMPUTATIONS

Our approach to diagnosing erroneous rules of signed-number operations is to generate a binary vector from a student's response by setting each element of the vector to 1 or 0 according to correct and erroneous actions taken by the student. The elements of the "error vector" comprise all possible operations in doing the problem. Determining the elements of the vector requires a careful examination of the logical flow of steps involved in reaching the answer to a given problem. Although it is not necessary to have a procedural network of the computational operations of signed-number arithmetic, all possible errors that might occur in students' reponses must be considered as the dimensions of the error vector. Selection of the dimensions is crucial in order to increase the capability of diagnosing misconceptions possessed by the student by the error vector system.

Instruction and error vectors: A 64-item free response test composed of addition and subtraction problems of signed-number computation (Appendix 1) was administered to 127 eighth graders of a junior high school, upon completion of three weeks of instruction. The instructional method and the students' performance over the three-week period were carefully observed and task analysis of the teachers' instruction was carried out. A logical flow chart of the task components was summarized in Figure 1.

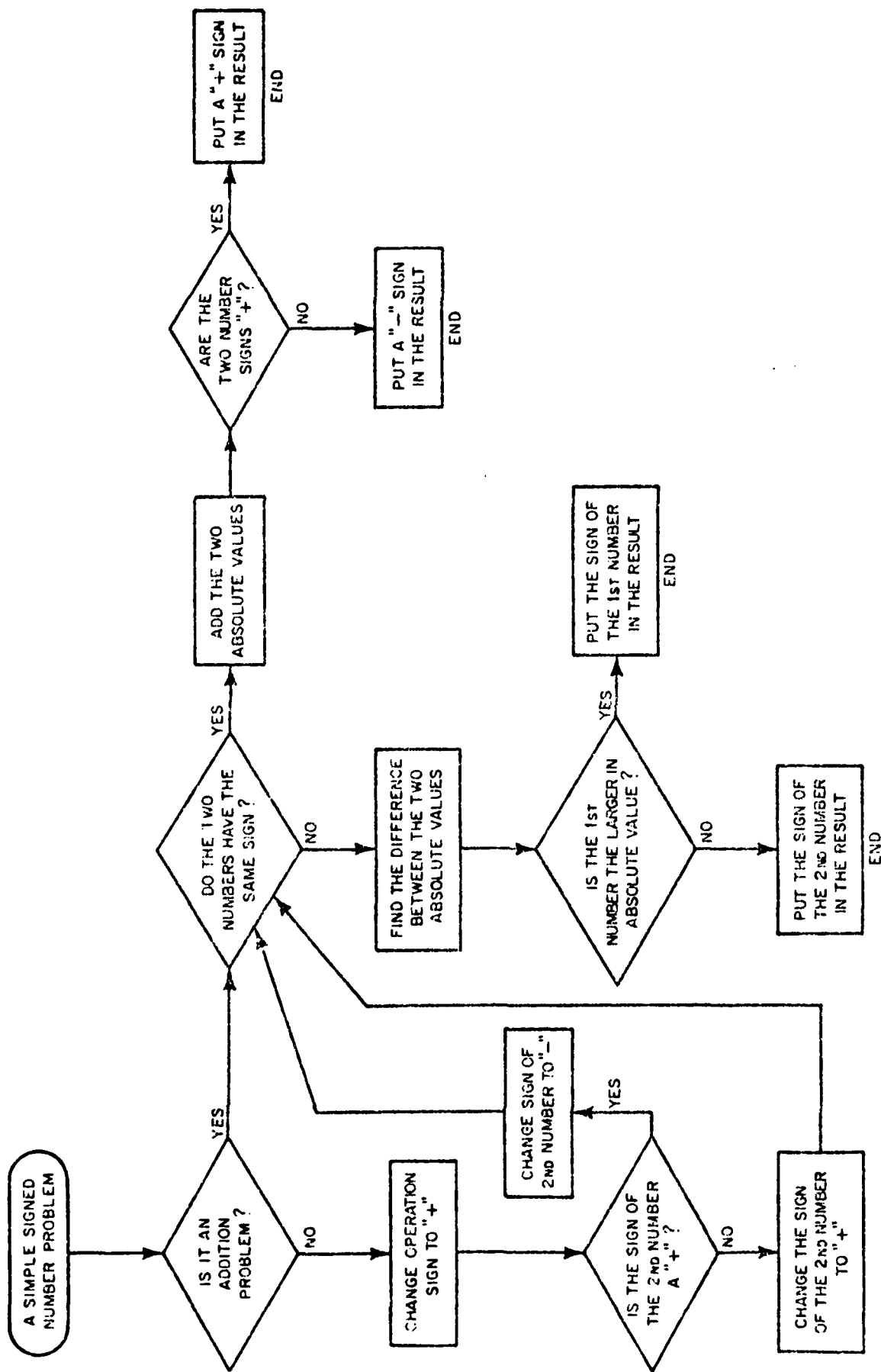Insert Figure 1 about here

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FIGURE 1: A FLOWCHART FOR SOLVING SIMPLE SIGNED NUMBER PROBLEMS ACCORDING TO THE TEACHING METHOD.

A main emphasis of the teacher's instructional method was to identify
the problem task first and apply an appropriate rule of operation to
the problem. The rules of operation consist of two different types
of task for addition problems and one extra, essential type for
subtraction problems.

In the addition problems, the student must identify whether or not
the two numbers have the same sign. If the signs of the two numbers
are the same, then they add the absolute values of the two numbers
and take the common sign of the two numbers for the answer. If the
sign of the two numbers are different, then they subtract the absolute
value of the smaller number from the absolute value of the larger
number and take the sign of the larger number. As can be seen in Figure 1,
the first decision to be made is which operation -- adding the two abso-
lute values or subtracting one from the other -- is to be done. A few
possible errors in making the decision can be involved. They are listed
in the upper half of Table 1 as the category of errors related to absolute-
value operations. The first digit of the error code is uniformly 1,
signifying that this is the first error vector. The second digit
denotes the element number.

Insert Table 1 about here

The elements of this error vector (referred to as the absolute-value
error vector hereafter) represent the three actions that may be taken,
and are 1 or 0 as follows;

Table 1

Elements of "Error Vectors" in the Addition of

Signed-Number Operation Such as a + -b

Category of Error	Error Code	Description of Operation
Related to	11	right operation
Absolute value	12	adding the absolute values of the two numbers
Operations	13	subtracting the absolute value of the smaller number from the absolute value of the larger number
Determining the Sign of Answers	21	sign of the larger number
	22	sign of the smaller number
	23	taking always a + sign
	24	taking always a - sign
	25	taking the sign of the first number
	26	taking the sign of the second number

$$\begin{bmatrix} x_1 \\ \\ \\ x_2 \\ \\ \\ \\ x_3 \end{bmatrix}$$

$x_1 \begin{cases} = 1 \text{ if the right operation is taken} \\ = 0 \text{ if not} \end{cases}$

$x_2 \begin{cases} = 1 \text{ if the absolute values of the two numbers} \\ \quad \text{are added} \\ = 0 \text{ if not} \end{cases}$

$x_3 \begin{cases} = 1 \text{ if the absolute values of the two numbers} \\ \quad \text{are subtracted} \\ = 0 \text{ if not} \end{cases}$

The second decision involved in the addition operation is to choose a right sign for the answer. If the signs of the two numbers are the same, the right sign for the answer is the common sign of the two numbers. But if the signs of the two numbers are different, then the sign of the larger number must be chosen. The six possible errors involved in choosing the sign for the answer are listed in the lower section of Table 1 (referred to as sign error vector hereafter). These elements are determined in the same manner as the absolute-value error vector.

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}
$$

$y_1 \begin{cases} = 1 \text{ if the sign of the larger number is taken} \\ = 0 \text{ if not} \end{cases}$

$y_2 \begin{cases} = 1 \text{ if the sign of the smaller number is taken} \\ = 0 \text{ if not} \end{cases}$

$y_3 \begin{cases} = 1 \text{ if } + \text{ sign is taken} \\ = 0 \text{ if not} \end{cases}$

$y_4 \begin{cases} = 1 \text{ if } - \text{ sign is taken} \\ = 0 \text{ if not} \end{cases}$

$y_5 \begin{cases} = 1 \text{ if the sign of the first number is taken} \\ = 0 \text{ if not} \end{cases}$

$y_6 \begin{cases} = 1 \text{ if the sign of the second number is taken} \\ = 0 \text{ if not} \end{cases}$

As mentioned earlier, the elements of the two vectors are determined by task analysis and the results of an error analysis performed on real data. There are more possible errors than those described in Table 1, but the mechanism of diagnosing misconceptions possessed by the student is illustrated by these nine elements of the two error vectors in order to simplify a complex algorithm on which the actual computer program of automated error analysis is based. New elements can be easily added to both the error vectors without causing much change in the computer algorithm. At the same time, elements that are not descriptive of actions taken by any student can be dropped so that the dimensionality of the vector can be reduced.

<u>Mechanism of error vector system</u>: Suppose a student responded to three
items as follows: -5 + 6 = +11, +7 + -9 = +16 and -3 + -4 = +7. For the
first item, -5 +6, each element of the 3 x 6 matrix product of the two
error vectors, the absolute value and sign error vectors, is produced
by following the specifications of the rules given in Table 1. Table 2
is the summary of responses to the item. Since the student's response is
+11, the cells (12,21), (12,23) and (12,26) in which +11 is seen, represent
the algorithms possibly used by the student. The response, +11, to the
problem may be obtained by adding the two numbers and taking the sign of
the larger number, always taking a + sign for the answer, or taking the
sign of the second number. This is equivalent to saying that the matrix

Table 2

Responses to Item -5 + 6 for 18 Events

(Algorithms), Binary Error Vectors and Their Matrix Product

Error Code	21	22	23	24	25	26
11	+1	-1	+1	-1	-1	+1
12	+11*	-11	+11*	-11	-11	+11*
13	+1	-1	+1	-1	-1	+1

The Matrix Product of Two Error Vectors

sign	
absolute value	1 0 1 0 0 1
0	0 0 0 0 0 0
1	1 0 1 0 0 1
0	0 0 0 0 0 0

product of two vectors (010) and (101001) yields 1 in the cells (2,1), (2,3) and (2,6). Leaving the three rules of operation as the student's possible algorithm, we go on to the second item to see if the rule of operation can be determined uniquely.

For the next item, +7 + -9 = +16, Table 3 shows the responses obtained by using the 18 possible algorithms. The cells (12,22), (12,23) and (12,25) contain the numbers equal to the student's answer, therefore his/her

Table 3

Responses to Item +7 + -9 for 18 Algorithms,

Binary Error Vectors and Their Matrix Product

Error Code	21	22	23	24	25	26
11	-2	+2	+2	-2	+2	-2
12	-16	+16*	+16*	-16	+16*	-16
13	-2	+2	+2	-2	+2	-2

The Matrix Product of two Error Vectors

sign	
absolute value	0 1 1 0 1 0
0	0 0 0 0 0 0
1	0 1 1 0 1 0
0	0 0 0 0 0 0

possible rules of operation are: Adding the two numbers and taking the
sign of the smaller number, always taking a + sign in the answer, or
taking the sign of the first number. In other words, the matrix product
of the two vectors, absolute-value and sign error vectors, (010) and (011010)
predicts these three possible rules of operation by yielding 1's in the
cells corresponding to these algorithms. If the student behaved consistently
for responding to the items, -5 +6 and +7 + -9, then elementwise multi-
plication of the two matrix products produces only one non-zero element,
thus signalling the only possible rule, (12,23) or adding the two numbers
and always taking a + sign for the answer. This rule yields a response,
+7 to the third item -3 + -4 which matches with the student's answer
for the item.

The procedures illustrated in the above example may be stated more
generally as follows. In order to determine a consistent rule of operation
uniquely, it is necessary to consider a set of appropriate types of items.
After the two binary vectors for each of several problems have been
generated from the student's responses to these problems, the elementwise
multiplication of all the vectors of each type, the absolute-value vector
and sign vector, respectively, produces a pair of error vectors for the
entire set of problems. If the student behaved consistently, applying
a single rule throughout the problems, there will be only one 1 in
each of the two error-vector products. Thus his/her consistent rule can
be determined uniquely. If the student does not behave consistently and
applies different rules of operation for each of the problems in the set,
then diagnosing his/her error will be very difficult by any method.

student   #2

teacher last year:

teacher this year:

score = 28

Task	Response	Time	Response	Time	Response	Time	Response	Time
1	14	25 sec	11	9 sec	8	8 sec	13	8 sec
2	-16 OK	10 sec	13	12 sec	10	6 sec	19	8 sec
3	15	13 sec	12	6 sec	21	17 sec	6	12 sec
4	11 OK	9 sec	15 OK	15 sec	12 OK	12 sec	14 OK	19 sec
5	15	8 sec	11	6 sec	17	12 sec	13	12 sec
6	10 OK	5 sec	18 OK	6 sec	13 OK	5 sec	17 OK	6 sec
7	14	9 sec	-12	10 sec	-6	7 sec	16	8 sec
8	-23	23 sec	-22	11 sec	-13	7 sec	-12	27 sec
9	-15 OK	10 sec	-10 OK	9 sec	-17 OK	6 sec	-16 OK	6 sec
10	-19 OK	8 sec	-11 OK	13 sec	-12 OK	8 sec	-18 OK	6 sec
11	-8	7 sec	-13	7 sec	-14	22 sec	-11	11 sec
12	16 OK	8 sec	10 OK	6 sec	11 OK	6 sec	-17	12 sec
13	-15 OK	8 sec	-13 OK	8 sec	-16 OK	7 sec	-10 OK	24 sec
14	-12 OK	8 sec	-14 OK	5 sec	-13 OK	5 sec	-15 OK	6 sec
15	-10	7 sec	-8	7 sec	-6	6 sec	-14	8 sec
16	-13	9 sec	-19	12 sec	-23	11 sec	-17	11 sec

Times for typing in before start of test:

-9	4.332	+10	5.328	+6	4.229	-13	4.164
-2	4.008	+ 8	4.250	+13	7.050	-4	3.341
+15	3.885	-7	3.166				

HELP1 to DELETE, NEXT for next student, BACK to exit

Figure 2: The Performance of Student #2

Table 4

Examples of Error Vectors for Addition Problems

Committed by Two Students in the November Data

Task #		Student #1		Response Pattern	Student #2		Response Pattern
6	6+4=10			1			1
15	-6+4=-2	101	100110*	1	010	100110	0
3	12+-3=9	101	101010	1	010	101010	0
5	-3+12=9	101	101001	1	010	101001	0
10	-14+-5=-19	110	110111	1	110	110111	1
11	3+-5=-2	101	100101	1	010	100101	0
14	-5+-7=-12	110	110111	1	110	100110	1
7	8-6=2	e2		0	14		0
8	-16-(-7)=-9	e2		0	-23		0
16	2-11=-9	e2	(e2,11,21)	0	-13	(e1,12, 21)	0
13	-3-+12=-15	e2		0	-15		1
1	-6-(-8)=2	e2		0	14		0
12	9-(-7)=16	e2		0	16		1
4	1-(-10)=11	e2		0	11		1
2	-7-9=-16	e2		0	-16		1
9	-12-3=-15	e2		0	-15		1

* "absolute value" and "sign" error vectors.

For Student #2, elementwise multiplication of the two sets of error vectors yields:

right	add two numbers	subtract two numbers		right	sign of minimum	+	-	sign of first	sign of second	
(0	1	0	)	(1	0	0	0	0	0	)

The rule of operation which student #2 used is that two numbers are always added and the right signs (sign of the larger number) were taken.

e2: Bring down the first number correctly, change the subtraction sign to + and does not change the sign of the second number

$$-16-(-7) \rightarrow -16+-7.$$

Examples: One of the most common errors in signed-number arithmetic is adding the two numbers and taking the sign of the larger number. Student #2 whose error vectors and response pattern are shown in the right-hand segment of Table 4, used this erroneous rule consistently for all six addition problems, with at least one negative number, that appear in the second column of the table. Figure 2 is a copy of the display on the PLATO system showing the performance of student #2 on the 64-item signed-number test given in Appendix 1. The binary error vectors of the addition problems shown in Table 4 indicate that all six absolute-value error vectors have 1 in the second column while 1 is in the first column of all sign error vectors. This means Student #2 committed the error described earlier in this example, and obtained the right answer to Task 10, -14 + -5, coincidentally. In order for us to determine this error uniquely, Student #2 needs to have taken one of the three pairs of Tasks, (15,5), (3,11) and (5,14). It is obvious that other combinations of tasks cannot provide a unique rule of the student's misconception by the error-vector system approach.

Efficient ways to choose the appropriate set of items so as to determine any consistent rules of operation uniquely by the error vector system are very crucial and will be investigated in a future study. This is especially necessary when the mechanism of the error vector system is applied for computerized adaptive testing. It is best to diagnose an error by giving the minimum number of items.

Insert Table 4 about here

Insert Figure 2 about here

Let us consider the six different types of addition problems and
denote them as follows:

A: $-$ larger number + smaller number = $-L+S$

B: $-$ smaller number + larger number = $-S+L$

C: smaller number + $-$ larger number = $S+-L$

E: larger number + $-$ smaller number = $L+-S$

E: $-$ smaller number + $-$ larger number = $-S+-L$

F: $-$ larger number + $-$ smaller number = $-L+-S$

The 18 "events", represented by the cells of the 3×6 matrix product
of the two binary error vectors, will uniquely determine the rule used
by the student when we form the elementwise product of such matrices for
an appropriate subset of the six problem types. The above example required
two problems, of the B and C types, to determine the student's rule uniquely.
By examining all combinations of error vectors, both the absolute value
and sign vectors. The Tasks A through F in addition operation have the
capability of determining and diagnosing consistent rules of operation
uniquely so long as the elements of the two vectors are limited to the
present specifications of nine types.

Adaptive testing: An adaptive test for signed-number addition problems
was programmed on the PLATO system and used with about 160 seventh graders
at the junior high school, who had just completed computerized instruction
on signed-number operations, and taken the 64-item free response linear
test. The adaptive test was designed to repeat a cycle of testing, diagnosing
and reviewing three times. After the first set of problems is analyzed by the
error vector system, the test program presents a statement telling the student
his/her erroneous rule and then giving him/her a brief review lesson of addition
problems. Upon completion of the review lesson, the second set of problems is adminis-

tered. If the student acquires the right rule (which the error vector system also judges from the responses to the items), then the testing will be terminated because the student has completed the program.

Table 5 is a copy of the display on the PLATO system after error analyses for 50 students had been completed. The numbers in the parentheses designate types of errors in each testing. Fifteen out of 50 students arrived at the right rule (11,21) and seven students mastered taking the right sign for the answer but failed to choose the right aboslute value operation consistently.

The first student, #1, has a pattern of (21)(21)(11,21). Interpretation of Code (21) with the description given in Table 1, taking the sign of the larger number in the answer, makes clear what happened with Student #1. He took the right sign for his answer but not the absolute values. After reviewing the instruction twice, he learned how to get the right absolute value for the answer.

Student #2 responded to the problems either randomly or using a rule not determined by the error vector system. But her second try is (23); in other words, she always takes the + sign in the answer. Diagnosis of her error type and reviewing the instruction were not effective for her.

Student #5 repeated (13,21) twice -- that is, subtracting two numbers and taking the sign of the larger number. But on the third try, his pattern changed to an entirely new rule. #10 repeated the same pattern as #5 twice, but the third pattern converged to the right operation.

_____
Insert Table 5 about here
_____

Table 6 shows the summary result of error analysis performed on the
64-item test. Since the linear test did not give any feedback during testing
time, students tended to hold their rule of operation throughout the
period even if it was wrong.

_____
Insert Table 6 about here
_____

ERROR VECTORS FOR SUBTRACTION PROBLEMS

The teachers at the junior high taught their students to solve
subtraction problems by converting them to addition problems. The rule
is that, *without changing the first number*, students must change the operation
sign - to +, then change the sign of the second number. By so doing, all
subtraction problems will be converted to addition problems. Let us denote
the subtraction tasks and an elementary addition task in the 64-item test
as follows:

P:	8-6	K:	1-(-10)
G:	2-11	L:	-6-(-8)
H:	-7-9	M:	-16-(-7)
I:	-12-3	N:	-3-+12
J:	9-(-7)	X:	6+4

There are three kinds of mistakes that students make when converting
a subtraction task to an addition problem. The first mistake is not changing
the sign of the second number (the code 110 was assigned to this operation).

Table 5

The Number Line Method
Error Analysis of Addition Problems in Adaptive Testing

Student #	Error Type	Student #	Error Type
1	(21)(21)(11,21)	26	(25) 0 0
2	0 (23)(23)	27	(13,26)(21)(13,21)
3	(24)(24)(24)	28	(13,24)(13,24)(25)
4	(26) 0	29	(11)(21)(11,21)
5	(13,21)(13,24)(26)	30	(13,23)(13,23)(11,21)
6	(23)(23) 0	31	(11,21)
7	(23)(11,26) 0	32	0 0 (13)
8	0 (24)(11,21)	33	0 0 (11,21)
9	(13)(13)(13,24)	34	0 0 0
10	(13,21)(13,21)(11,21)	35	(13,23)(23)(13,23)
11	0 (21)	36	(11,21)
12	0 (25)	37	0 (13)
13	0 0 (13,24)	38	(21)(21)
14	(24)(24)(12,24)	39	(21)(13,21)(21)
15	0 (25)(13,24)	40	(21)(13,21)(21)
16	(13,21)(21)(23)	41	(23)(23)(23)
17	(11,24)(24)(21)	42	0 0 0
18	(24)(11,21)	43	0 (21)(11,21)
19	0 0 (24)	44	(13) 0 (21)
20	0 (25)(25)	45	0 (11,21)
21	(12,24)(12,24)(21)	46	(25)(24)(12,24)
22	(13,23)(11,26)(13,26)	47	(24)(21)(24)
23	(12)(11,21)	48	0 (11,24)(11,21)
24	0 (11,21)	49	(11,21)
25	0 (23) 0	50	0 (24)(24)

Table 6

Error Types Over Four Parallel Subtests of the First

Test in Group 1 (Addition Problems)

	Less Consistent Errors			More Consistent Errors	
S#	Error Type	Score	S#	Error Type	Score
40	(11,21)(11,21)(11,21)(11,21)	19	3	(11,21)(11,21)(11,21)(11,21)	20
52	(21)(11,21)(11,21)(11,21)	19	4	(11,21)(11,21)(11,21)(11,21)	20
17	(11,21)(13,21)(21)(11,21)	18	13	(11,21)(11,21)(11,21)(11,21)	20
79	(13,21)(11,21)(11,21)(11)	18	34	(11,21)(11,21)(11,21)(11,21)	20
12	(11)(11)(11,21) 0	16	65	(11,21)(11,21)(11,21)(11,21)	20
45	(13,21)(11,21) 0 (21)	16	27	(11,21)(11)(11,21)(11,21)	19
55	(11)(11,21)(21)(11)	16	35	(11)(11,21)(11,21)(11,21)	19
31	(11)(13)(13,21)(11,21)	15	36	(11,21)(11)(11,21)(11,21)	19
44	(11,21)(24)(21)(13,21)	15	41	(13,21)(11,21)(11,21)(11,21)	19
73	(13)(11,21)(11)(11)	15	42	(13,21)(11,21)(11,21)(11,21)	19
17	(11,(13,21)(13)(13)	14	58	(13,21)(11,21)(21)(11,21)	18
20	(13,21)(21) 0 (13,21)	14	37	(13)(11,21)(11)(11)	17
70	(21) 0 (21) 0	14	50	(13,21)(0)(11,21)(11,21)	17
72	(13,21)(13,21) 0 0	14	29	(13,21)(13,21)(21)(13)	14
30	0 (21) 0 (13,21)	13	53	(11,21)(11,21) 0 0	14
60	(11)(13) 0 0	13	2	0 (25)(13,25)(13,25)	10
57	0 (13)(13,21)(13)	12	22	0 (13,24)(13)(13)	10
18	(13) 0 (13)(13,23)	10	48	(13,21)(13,24)(13,24)(13,24)	10
10	0 0 (13) 0	9	46	0 0 (13,23)(23)	9
23	(24) 0 (13,24)(24)	9	56	(11)(21)(23) 0	9
1	0 (13,24)(13,24)(24)	8	26	(25)(13,25)(13,25) 0	8
7	(13,24) 0 0 0	8	76	(23)(13,23)(13,23)(13,23)	8
8	(13)(13)(13)(13)	8	16	0 (26) 0 0	7
54	(13,24) 0 (21)(25)	8	43	0 (13)(13)(13))	7
59	0 0 0 0	7	24	(13,26)(13,26)(13,26)(13,26)	5
5	0 (13,23) 0 (13,23)	6	21	(24)(24)(12,25)(12,25)	4
67	(22)(23) 0 0	6	25	0 0 0 0	4
9	(13,24)(24)(12,24)(12,24)	4	49	(12)(12)(12,24)(12,24)	4
47	(22)(13)(23)(13)	4	64	0 (12,23)(12,23) 0	1
14	0 0 0 0	3			
69	0 0 0 (12,23)	3			
19	(23)(12,23)(23)(12,23)	2			
39	(24)(24)(24)(12,24)	2			
62	0 (24)(24)(24)	2			
71	0 0 (24) 0	2			
6	0 (21) 0 0	1			

The second mistake is changing the signs of both numbers (referred to by Code 011 hereafter). The third mistake is changing the sign of the first number instead of the sign of the second number (referred to by Code 010 hereafter). The right conversion is denoted by Code 111. Note that these codes are _not_ error vectors. The converted tasks of the problem, 10 - (-1), by using each of the four types are listed below:

Code	Subtraction	Converted Addition	
110	10-(-1)	10+(-1)	D type
011	10-(-1)	-10+(+1)	A type
010	10-(-1)	-10+(-1)	F type
111	10-(-1)	10+(+1)	X type

Table 7 is a summary list of addition tasks (upper half of the table), the original subtraction tasks along with their converted addition tasks. As can be seen easily, each problem of subtraction has four possible converted tasks according to the types of conversion 111, 110, 011 and 010, the first being the correct conversion.

Insert Table 7 about here

Each of the four converted tasks has a pair of error vectors, the absolute-value and sign vector, generated from the student's response.

Suppose a student answers the item 7 - 8 = ? and gets the answer -1. Then we will have four possible pairs of absolute-value and sign error vectors as shown in the last two columns of Table 8. Each of these pairs represents the operations that must be carried out on the converted addition problem of that row in order to produce the result -1.

Table 7

A List of Converted Tasks of Subtraction to

Addition Problems by Four Possible Ways

Name of Task	Type of Task
A	$-L^{*}+S^{**}$
B	$-S+L$
C	$S+-L$
D	$L+-S$
E	$-S+-L$
F	$-L+-S$
X	Positive + Positive

Original Tasks		Converted Tasks			
Name of Task	Type of Task	111	110	011	010
G	$S-L$	C	X	E	B
H	$-S-L$	E	B	C	X
I	$-L-S$	F	A	D	X
J	$L-(-S)$	X	D	A	F
K	$S-(-L)$	X	C	B	E
L	$-S-(-L)$	B	E	X	C
M	$-L-(-S)$	A	F	X	D
N	$-S-(+L)$	F	B	C	X
O	$-L-+S$	F	A	D	X

* The larger number of the two numbers in absolute value

** The smaller number of the two numbers in absolute value

Table 8

Four Possible Conversions of Task 7 - 8 to

the Addition Problems

Response	Code	Type of Task	Absolute Value Vector	Sign Vector
-1	111	7+-8, C	(1 0 1)	(1 0 0 1 0 1)
-1	110	7+8, X	(0 0 1)	(0 0 0 1 0 0)
-1	011	-7+-8, E	(0 0 1)	(1 1 0 1 1 1)
-1	010	-7+8, B	(1 0 1)	(0 1 0 1 1 0)

If the student knows the right rule of operation for addition problems,
then he/she should be able to carry out the converted task correctly.
Since, of the four pairs of error vectors, only the pair in the first row
represents the correct operation on the converted addition task, it may be
concluded that this student used the conversion shown in the first row,
namely, the correct conversion (111).

More generally, in the example of Table 8, the student's rule of converting
subtraction to addition is the Code 111 type if he/she can correctly answer
the Task C addition problem with the right rule. If, on the other hand,
the student's rule of operation for addition problems is "the right rule
for taking the absolute value but taking the sign of the first number in
the answer" then the student's conversion rule of subtraction to addition
is the Code 010 type.

The two examples in Table 4 are the eighth graders who took the 64-item
signed-number test after the three weeks of instruction and practice in
signed-number operations. Student #1 uses the right rule for addition

problems but converts subtraction into addition incorrectly. His rule is expressed by Code 110. We designated his rule by e2 in the table but it means that he committed the wrong conversion rule, 110 and applied the right rule of addition to the newly converted addition tasks. Student #2 converted the subtraction problems correctly, but her addition rule was wrong: Adding the two absolute values and taking the sign of the larger number. Thus, her rule was coded (e1,12,21). Tables given in APPENDIX 2 show the binary error vectors for Student #1, and APPENCICES 3 and 4 are the performance of the test and binary error vectors for Student #2. Further discussion of the topic "Error Vectors for Subtraction Problems" will be given in a subsequent technical report.

DISCUSSION

A brief description of the method using what we termed binary error vectors for diagnosing misconceptions possessed by students was given and its mechanism was illustrated with several examples obtained from real data. A computer program capable of finding 240 basic errors in signed-number computations -- subtracting or adding two absolute values, taking a particular sign for the answer and converting subtraction to addition problems -- has been written on the PLATO system and used for analyzing the 64-item conventional test as well as the adaptive test of addition problems. The results of error analyses performed on over two hundred students will be discussed in detail in other technical reports, because the findings should be of interest to researchers in fields as diverse as psychometrics,

instructional design and learning theory, thus making it desirable to prepare several technical reports with different foci.

The actual computer algorithm is more complicated than it was feasible to describe in this paper, but the basic mechanism is the same as illustrated in the examples. Although the error-vector system was discussed in the context of signed-number operations, the rationale and technique of the method can be easily expanded to other areas of arithmetic, algebra, or even natural-language syntax. Error components (or generic type of error vectors such as the absolute-value error vector or the sign error vector), elements of each error vector can be determined by performing a careful task analysis on the subject area of interest. Great caution must be taken in the analysis because results of our preliminary data analyses showed that different instructions yielded different kinds of erroneous rules. This implies that the logical flow of task components depends on the type of instructional methods that have adopted different conceptual frameworks to teach the same topic.

For example, Greeno (1976) showed two different procedural representations of the concept (or procedure) of adding two fractions. One is the procedure of finding equivalent fractions, using a special representation of fractional quantities and the other is using a set-theoretic representation of fractional quantities. The two approaches use different methods in conceptualizing the same task. Our question is whether or not different instructional approaches yeild different types of erroneous rules. This question was answered in the affirmative by our experimental study.

The experiment that was conducted last January -- by which the relation-
ships between characteristics of achievement data and their progressive
changes in various learning stages given different instructional methods
have been investigated -- revealed that two different instructional methods,
using the number line and verbal stories to carry out computations of
signed-number, yielded quite a few different types of errors. The results
of error analyses regarding this aspect will be discussed in a future tech-
nical report.

This finding raises a question as to the extent to which our present
error vector system is valid for the number line method, since it was con-
structed for the verbal rules of operations and stories approach. Further
investigation on this matter is underway.

The adaptive test for addition problems was designed and administered
to about 150 students on the PLATO system, although evaluation of the new
testing prodedure and its effect on teaching were not discussed in this report.
Selecting the minimum number of items from an item pool to determine a
consistent rule of operation for addition problems was not much trouble,
because there were not so many types of error. But a diagnostic adaptive
test for subtraction problems is not as easy to construct as one for addition
problems. As mentioned earlier in the report, the number of possible con-
sistent erroneous rules for subtraction problems increases drastically to
nearly 300. Thus, some new technology will be needed to allow systematic
and economical item selection procedure in order to determine any consistent
rules of operation. "BUGGY" by Brown and Burton apparently has its testing

procedure based on "the universal process network" of whole-number arithmetic. As our experimental study demonstrated, different instructional methods of teaching singed-number operations produced different types of erroneous rules; moreover the task components of each method for solving a given problem differ considerably. Although Brown & Burton discuss the importance of the procedural network to develop a diagnostic adaptive test, there cannot be a unique, universal procedural network for signed-number operations.

The item selection procedure used in our adaptive test for addition problems did not require a procedural network. Instead, the approach taken was quite similar to our method for determining the specific wrong rule of operation (if any) by means of the error-vector system. When a student gives a certain answer (either correct or incorrect) to the first item administered, each of the two error vectors (for the absolute value and the sign respectively) will generally contain several 1's, indicating that there are several alternative rules that he/she may have used in getting that answer. The next item to be administered is chosen so as to eliminate one or more of these 1's in the elementwise product of each of the two successive pairs of error vectors (the absolute-value error vectors of the two items, and the two sign vectors, respectively). Similarly, the next item is chosen to further eliminate 1's in the cumulative elementwise products, and so on until only one 1 remains in each of the two cumulative products. It can be shown that, provided a consistent rule of operation is always used, at most four items will suffice to uniquely determine the rule. Once the rule is determined and if it is found to be erroneous, a diagnostic statement is made to the student, and

he/she is given a review sequence as described on p. 17. If there is a switch of rules from item to item, the errors are judged to be "random", and no diagnosis is made. This outcome is indicated by a 0 in Tables 5 and 6.

A philosophical consideration is relevant at this point. Namely, is it really meaningful to use the same adaptive test for the dual purpose of measuring a student's achievement level on the one hand and diagnosing his/her misconceptions on the other? In our case, we treated addition and subtraction problems separately, and administered an adaptive test only in addition in the study described in this report. The reasons for this were twofold: Firstly, the wrong rules that were used consistently for addition and those occurring in subtraction were considerably different; secondly, the values of the ICC parameter _b_ were substantially different between addition and subtraction problems. Our intention was to use this adaptive test in addition for the dual purpose mentioned above. However, as can be seen from Table 5, most students were still committing consistent errors at the time the test was administered, and hence it was not very effective for measuring their achievement level. Only when the students approach mastery, resulting in a reduction of the incidence of consistent errors and a relative increase of random errors due to typing mistakes, momentary lapses of attention and the like, does it seem meaningful to measure their achievement level. An exception to this dictum may occur when it is desired to investigate how consistent errors change as learning proceeds. An experiment is underway- and results of data analyses that may throw some light on this question will be discussed in a subsequent Technical Report.

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Appendix 1

The Signed-Number Test

I		**II**		**III**		**IV**	

1. -6-(-8)=2	17. -1-(-10)=9	33. -3-(-5)=2	49. -2-(-11)=9
2. -7-9=-16	18. -2-11=-13	34. -4-6=-10	50. -5-14=-19
3. 12+-3=9	19. 7+-5=2	35. 15+-6=9	51. 4+-2=2
4. 1-(-10)=11	20. 3-(-12)=15	36. 5-(-7)=12	52. 6-(-8)=14
5. -3+12=9	21. -1+10=9	37. -4+13=9	53. -2+11=9
6. 6+4=10	22. 10+8=18	38. 2+11=13	54. 4+13=17
7. 8-6=2	23. 7-5=2	39. 4-2=2	55. 9-7=2
8. -16-(-7)=-9	24. -12-(-10)=-2	40. -11-(-2)=-9	56. -7-(-5)=-2
9. -12-3=-15	25. -6-4=-10	41. -13-4=-17	57. -9-7=-16
10. -14+-5=-19	26. -10+-1=-11	42. -7+-5=-12	58. -10+-8=-18
11. 3+-5=-2	27. 2+-11=-9	43. 6+-8=-2	59. 1+-10=-9
12. 9-(-7)=16	28. 6-(-4)=10	44. 10-(-1)=11	60. 13-(-4)=17
13. -3-+12=-15	29. -2-+11=-13	45. -7-+9=-16	61. -4-+6=-10
14. -5+-7=-12	30. -6+-8=-14	46. -2+-11=-13	62. -3+-12=-15
15. -6+4=-2	31. -5+3=-2	47. -4+2=-2	63. -8+6=-2
16. 2-11=-9	32. 5-14=-9	48. 7-16=-9	64. 4-13=-9

Item	Response	Error Vector
3	9	101 101010
5	9	101 101001
6	10	110 111011
10	-19	110 110111
11	-2	101 100101
14	-12	110 110111
15	-2	101 100110
1	-14	010 010110, 110 110111, 110 000100, 010 100101 e2
2	2	001 001000, 001 001000, 101 101001, 001 111011 e2
4	-9	001 000100, 101 100101, 101 010010, 001 110111 e2
7	14	010 101010, 110 111011, 110 001000, 010 011001 e2
8	-23	010 100110, 101 110111, 110 000100, 010 010101 e2
9	-9	001 110111, 101 100110, 101 010101, 001 000100 e2
12	2	001 111011, 101 101010, 101 011001, 001 001000 e2
13	9	001 001000, 010 101010, 101 010010, 001 110111 e2
16	13	010 011010, 110 001000, 110 001000, 010 101001 e2
3	2	101 101010
5	-9	101 010110
6	18	110 111011
10	-11	110 110111
11	-9	101 100101
14	-14	110 110111
15	-2	101 100110
1	-11	010 010110, 110 110111, 110 000100, 010 100101 e2
2	9	001 001000, 101 101010, 101 101001, 001 111011 e2
4	-9	001 000100, 101 100101, 101 010010, 001 110111 e2
7	12	010 101010, 110 111011, 110 001000, 010 011001 e2
8	-21	000 100110, 000 100110, 000 000100, 010 010100 e0
9	-2	101 110111, 101 100110, 101 010101, 001 000100 e2
12	2	001 111011, 101 101010, 101 011001, 001 001000 e2
13	9	001 001000, 010 101010, 101 010010, 001 110111 e2
16	19	010 011010, 110 001000, 110 001000, 010 101001 e2

APPENDIX 2

```
1.   2  [-14]  010 010110, 110 110111, 010 010100, 010 100101 e=2
2.  -16 [2]    001 001000, 101 101001, 101 011010, 001 111011 e=2
3.   9  [9]    101 101010               101 010010
4.   11 [-9]   001 000100, 101 100101, 101 010110, 001 110111 e=2
5.   9  [9]    101 101001              101 101001
6.   10 [10]   101 001101
7.   2  [14]   010 101010, 110 111011, 110 001000, 010 011001 e=2
8.  -9  [-23]  010 100110, 110 110111, 101 000100, 010 010101 e=2
9.  -15 [-9]   001 110111, 101 100110, 101 010101, 001 000100 e=2
10. -19 [-19]  110 110111  110 111111
11. -2  [-2]   101 001101
12. 16  [2]    001 111011, 101 010010, 101 011001, 001 010000 e=2
13. -15 [9]    001 001000, 101 010001, 101 111011, 001 111011 e=2
14. -12 [-12]  110 110111
15. -2  [-2]   101 100110
16. -9  [13]   010 010110, 110 111011, 110 001000, 010 101001 e=2
1.   9  [-11]  010 010110, 110 110111, 110 001000, 010 100101 e=2
2.  -13 [9]    001 001000, 101 010001, 101 011010, 001 111011 e=2
3.   2  [2]    101 101010
4.   15 [-9]   010 001100, 101 001101, 101 010110, 001 110111 e=2
5.   9  [-9]   101 010110
6.   10 [18]   101 010110
7.   2  [12]   010 101010, 110 111011, 110 001000, 010 011001 e=2
8.  -2  [-21]  000 100100, 100 110111, 000 001001, 000 010101 e=0
9.  -10 [-2]   101 110111, 101 000110, 101 010101, 001 000100 e=2
10. -11 [-11]  110 110111
11. -9  [-9]   101 100101
12. 10  [2]    001 111011, 101 010010, 101 011001, 001 010000 e=2
13. -13 [9]    001 001000, 101 010001, 001 111011, 001 111011 e=2
14. -14 [-14]  110 110111
15. -2  [-2]   101 001101
16. -9  [19]   010 010110, 110 111011, 110 011000, 010 101001 e=2
```

APPENDIX 3

```
 1.   2   [14]    010 101001,  110 001000,  110 111011,  010 011010  e=3
 2. -16   [-16]   110 110111,  010 010110,  010 010101,  010 010101  e=1
 3.   9   [15]    010 101010   010 101010
 4.  11   [11]    110 111011,  010 011010,  010 101001,  110 001000  e=1
 5.   9   [15]    010 101001   010 101001
 6.  18   [18]    010 101001
 7.   2   [14]    010 101010,  110 111011,  110 001000,  010 011001  e=2
 8.  -9   [-23]   010 100110,  010 110111,  010 100100,  010 011001  e=2
 9. -15   [-15]   110 110111,  110 100111,  010 010101,  110 001100  e=1
10. -19   [-19]   110 110111   110 110111
11.  -2   [-8]    010 100101   010 010101
12.  16   [16]    110 111011,  010 101001,  010 011001,  110 010100  e=1
13. -15   [-15]   010 010101,  010 010110,  010 001100,  110 001100  e=1
14. -12   [-12]   110 110111
15.  -2   [-18]   010 100110   010 000110
16.  -9   [-13]   010 100101,  110 001000,  010 100100,  010 010110  e=3
 1.   9   [11]    010 101001,  010 101001,  110 111011,  010 010110  e=3
 2. -13   [13]    010 001000,  110 001000,  110 111011,  010 110110  e=4
 3.   2   [12]    010 101010   010 010100
 4.  15   [15]    010 111011,  010 100101,  010 101001,  110 001000  e=1
 5.   9   [11]    010 101001   010 101001
 6.  18   [18]    010 101001
 7.   2   [-12]   010 100100,  110 110111,  110 001000,  010 100110  e=3
 8.  -2   [-22]   010 100110,  110 101111,  010 101010,  010 010101  e=2
 9. -18   [-18]   010 100110,  010 101111,  010 010101,  010 001100  e=1
10. -11   [-11]   110 101111   110 001010
11.  -9   [-13]   010 001000   010 100101
12.  18   [18]    110 111011,  010 101001,  010 011001,  110 010000  e=1
13. -13   [-13]   010 010101,  110 001000,  010 010101,  110 000100  e=1
14. -14   [-14]   110 101111
15.  -2   [-8]    010 100110   010 010110
16.  -9   [-19]   010 100101,  110 110111,  010 101111,  010 010110  e=3
```

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